R.C.C Structures

Design Concepts

There are three design methods viz. working stress method, ultimate load method and limit state method. From 1970 onwards limit state method is popular because of its rational approach. IS: 456-2000 for the design of reinforced concrete (R.C.C.) structures is based in limit state design philosophy. The working stress method is fast disappearing from practice. Clause 18.2.2 of IS: 456-2000 requires that working stress method may be used only if it is not possible to adopt the limit state design method.

LIMIT STATE METHOD

Limit State of Collapse

This state corresponds to the maximum load carrying capacity. This limit may be correspond to: (i) flexure (ii) compression (iii) shear, and (iv) torsion.

Limit State of Serviceability

This limit corresponds to development of excessive deformation. This limit may be correspond to: (i) deflection (ii) cracking, and (iii) vibration

Building Code

A reinforced concrete structure should confirm to certain minimum specifications with regard to design and construction. The Indian Standards Institution issues building code requirements from time to time. The most recent code is the **code of practice for plain and reinforced concrete (IS:456 - 2000)**.

Partial Safety Factors For Loads

Under Limit State of Collapse

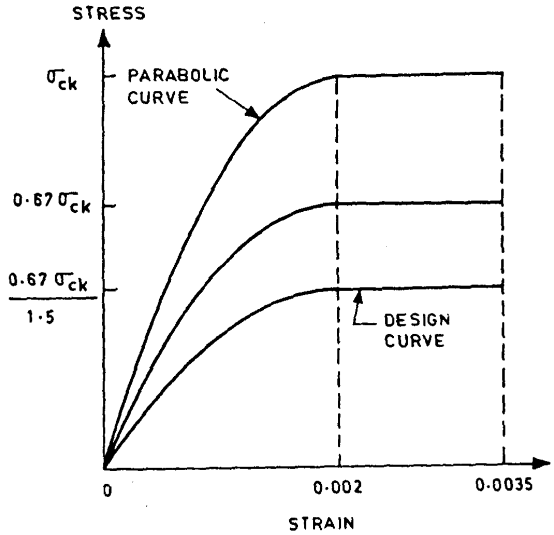
Load combination	Dead load, DL	Live load, LL	Wind load, WL
DL + LL	1.5	1.5	---
DL + WL	1.5	---	1.5
DL + LL + WL	1.2	1.2	1.2

Under Limit State Of Serviceability

Load combination	DL	LL	WL
DL + LL	1.0	1.0	---
DL + WL	1.0	---	1.0
DL + LL + WL	1.0	0.8	0.8

Stress - Strain Relationship For Concrete

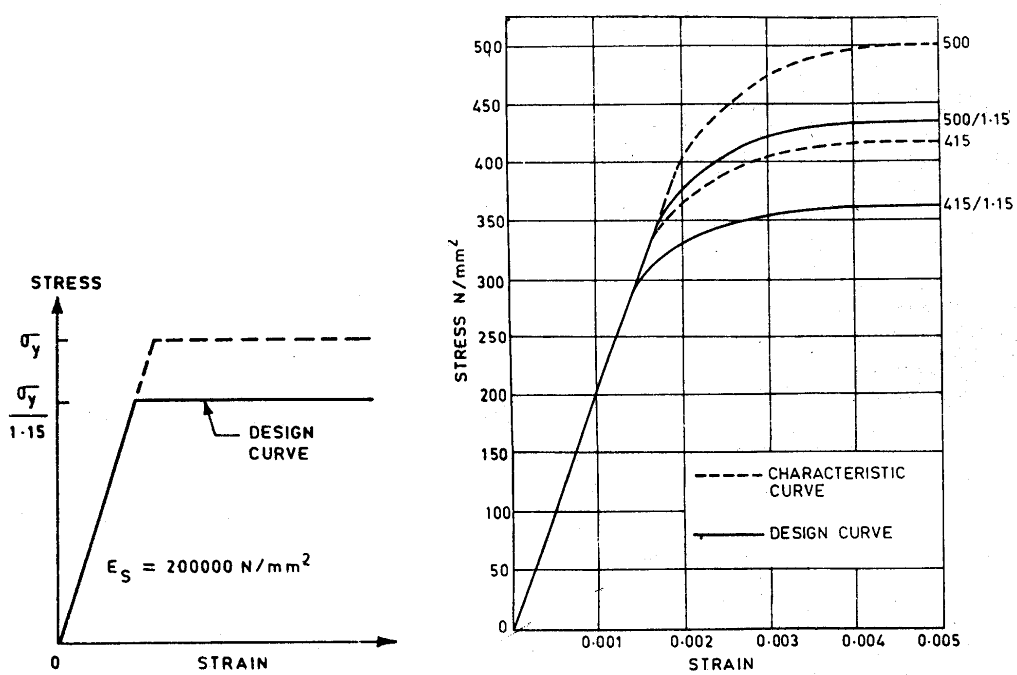
The code has idealized stress strain curve for concrete as shown in given figure.



For design purpose, the compressive strength of concrete in the structure is taken as 0.67 times the characteristic strength (σ_{ck}). The partial safety factor μ_m (= 1.5) is applied in addition to this factor.

Stress - Strain Relationship For Steel

For mild steel, the stress is proportional to strain upto yield point and thereafter the strain increases at constant stress as shown in given figure.



(a) Stress strain curve for mild steel (b) Stress strain curve for high strength deformed bars

For mild steel, the change from elastic to plastic condition is abrupt, whereas for high strength deformed bars, the change is quite gradual.

Beams

We know that concrete is fairly strong in compression but very weak in tension. Thus, the tensile weakness of concrete is overcome by the provision of reinforcing steel in the tension zone round the concrete to make a reinforced concrete (RC) beam.

There are three types of reinforced concrete beams :

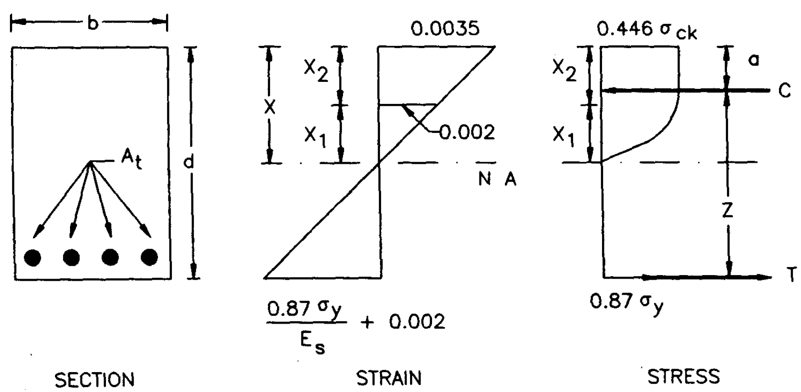
- Singly reinforced beams
- Doubly reinforced beams
- Flanged beams

In singly reinforced simply supported beams reinforcing steel bars are placed near the bottom of the beam where they are most effective in resisting the tensile bending stresses. In singly reinforced cantilever beams reinforcing bars are placed near the top of the beams.

SINGLY REINFORCED BEAMS

Assumptions

1. Plane sections normal to the axis remain plane after bending,
2. The maximum strain in concrete at the outermost compression fibre is taken as 0.0035 for any strength of concrete.
3. The relationship between stress - strain distribution in concrete is assumed to be parabolic as shown in given figure The maximum compressive stress is equal to **0.446** σ_{ck}**.**



4. The tensile strength of concrete is ignored,
5. The stress in reinforcement is derived from the representative stress strain curve for the type of steel used.
6. The maximum strain in tension reinforcement in the section at failure should not be less than following :

$$\varepsilon_s \geq \frac{\sigma_y}{1.15.E_s} + 0.002$$

where σ_y = characteristic stress in steel, E_s = modulus of elasticity of steel and ε_s = strain in steel at failure

Stress Block Parameters

With reference to figure given above, we see that

Depth of parabolic portion

$x_1 = 4x/7$

Depth of rectangular portion

$x_2 = 3x/7$

Design force of compression

$C = 0.36\,\sigma_{ck}.b.x$

Distance between top fibre and compressive force C would be

$a = 0.42x$

where x = depth of neutral axis from the extreme compression fibre, b = breadth of section, d = effective depth of section

Force of compression = force of tension

$0.36\,\sigma_{ck}.b.x = 0.87\,\sigma_y.A_t$

i.e. $x = \dfrac{0.87.\sigma_y A_t}{0.36\sigma_{ck}.b}$ where A_t = area of tension steel

Lever arm $z = d - 0.42\,x = d - 0.42\left\{\dfrac{0.87.\sigma_y A_t}{0.36\sigma_{ck}.b}\right\} = d - \dfrac{\sigma_y A_t}{\sigma_{ck} b}$

Modes of Failure

- **Balanced Reinforced Beam** If the ratio of steel to concrete in a beam is such that **maximum strain** in the two materials reach simultaneously, such a section is called **balanced section.**
- **Under Reinforced Beam** When the amount of steel is kept less than that in balanced section, it is called **under reinforced beam.** Under increasing bending moment, steel is strained beyond the yield point and the maximum strain in concrete remains less than 0.0035. Once steel has yielded, it does not take any additional stress for the additional strain and total force of tension remains constant. However, compressive stresses in concrete do increase with the additional strains. This increase continues until maximum strain in concrete reaches its ultimate value(0.0035), and concrete crushes. This is called tensile failure.
- **Over Reinforced Beam** When the amount of steel is kept more than that in the balanced condition, strain in steel remains in the elastic region. With increasing stress, strain in concrete reaches its ultimate value(0.0035), it crushes. The steel is still well within the elastic limit. This is called compression failure.

Moment of Resistance

Moment of resistance with respect to concrete

= compressive force x lever arm = $0.36\sigma_{ck}bxz$

Moment of resistance with respect to steel

= tensile force x lever arm = $0.87\sigma_y A_t xz$

Maximum Depth of Neutral Axis (x_m)

The maximum depth of neutral axis is limited to ensure that tensile steel will reach its yield stress ***before concrete fails in compression*.** Thus a compression failure is avoided. The values of x_m for three types of steel are given in table, below.

σ_y(N/mm^2)	x_m
250	0.53d
415	0.48d
500	0.46d

Limiting values of moment of resistance with respect to concrete and with respect to steel can be obtained by replacing **x** with $\mathbf{x}_m$ in moment of resistance equations.

Effective Span(l)

Simply supported beam or slab

$l = L_c + d$ where L_c = clear span and d = effective depth

Continuous beam/slab (if width of support ≤ $L_c/12$)

Same as in above equation.

How to Find Moment of Resistance(See table)

S. No.	Under Reinforced Section, $x < x_m$	Balanced section $x = x_m$	Over Reinforced section, $x > x_m$
1.	$x = \dfrac{0.87\sigma_y A_t}{0.36\sigma_{ck} b}$	$x = x_m$	$x = x_m$
2.	$z = d - 0.42x$	$z = d - 0.42x_m$	$z = d - 0.42x_m$
3.	$M_u = 0.87\sigma_y A_t z$	$M_u = 0.36\sigma_{ck} b x_m (d-0.42x_m)$ Or $M_u = 0.87\sigma_y A_t (d-0.42x_m)$	$M_u=0.36\sigma_{ck} b x_m (d-0.42x_m)$

Reinforcement Details

Minimum and maximum tension reinforcement

Minimum Tension reinforcement Area (A_0)

$A_0/bd = 0.85/\sigma_y$ where σ_y = characteristic strength of steel in N/mm^2

Maximum Steel Area (A_{max})

$A_{max} \le 0.04$ x cross sectional area i.e. $A_{max} \le 0.04.b.D$

Nominal cover to reinforcement

Nominal cover is the design thickness of concrete cover to all steel reinforcements. The actual cover *at site* should not be less than nominal cover plus 10 mm. The nominal cover for various exposure conditions to atmosphere are given below:

Exposure	Cover (mm)

Mild	25
Moderate	30
Severe	45
Very severe	50
Extreme	75

Spacing of reinforcement

The *horizontal* distance between two parallel main reinforcing bars should not be less than the greatest of the following :

- the diameter of the bar if the diameters are equal
- the diameter of the larger bar if the diameter are unequal, and
- 5 mm more than the nominal maximum size of coarse aggregate.

The *vertical* distance between two parallel main reinforcing bars should not be less than the greatest of the following :

- 15 mm
- the diameter of the larger bar if the diameters are unequal, and
- two thirds the nominal maximum size of the coarse aggregate.

Side face reinforcement

If depth of the web in a beam exceeds 750 mm, side face reinforcement should be provided along the two faces. The total area of such reinforcement should not be less than 0.1 % of the web area. It should be equally distributed on each of the two faces. The spacing of such reinforcement should not exceed 300 mm or web thickness, whichever is less.

DOUBLY REINFORCED BEAM

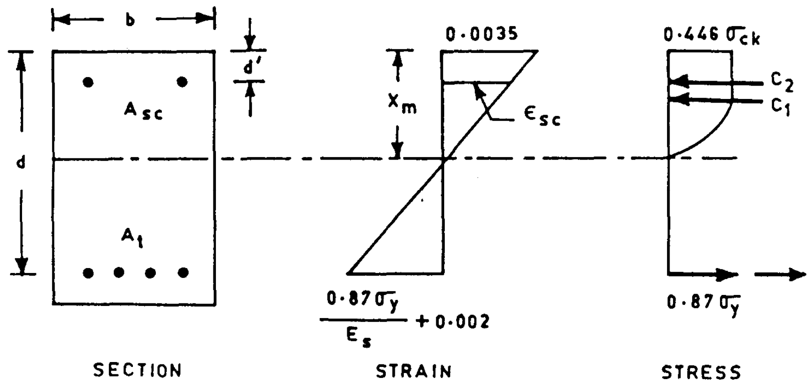
A doubly reinforced concrete beam is reinforced in both compression and tension regions. The necessity of using compression steel arises when the depth of the beam is restricted. In this condition, the strength of singly reinforced beam is inadequate.

Stresses in Tension and Compression Steel

- Tension reinforcement is designed for a stress of $0.87\sigma_y$.
- Design stress for compression steel corresponds to the strain in concrete at that level as shown in figure. Strain at the level of compression reinforcement (A_{sc}) is equal to

$$\varepsilon_{sc} = 0.0035\left(1 - \frac{d'}{x_m}\right)$$

Stress-strain relationship in a doubly reinforced beam is shown in given figure.



Knowing the strain at the level of compression steel, the stresses in steel can be obtained either from stress-strain curves for steels *or* from following table.

Stress σ_{sc} in Compression Steel

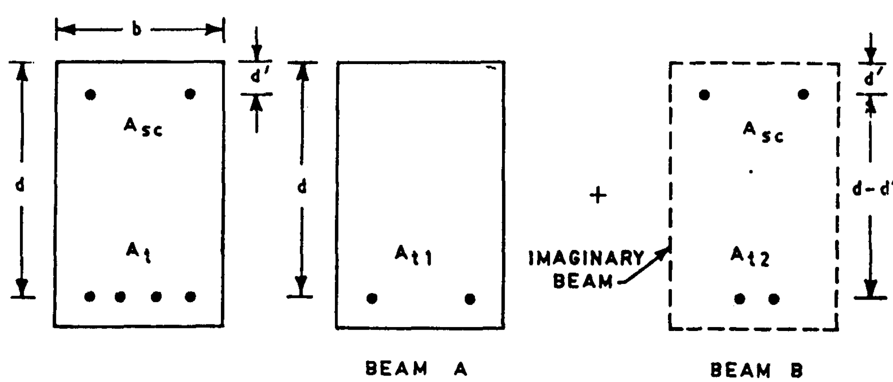
Grade of steel σ_y, N/mm^2	d'/d			
	0.05	**0.10**	**0.15**	**0.20**
250	217	217	217	217
415	355	353	342	329

Reinforcement Details

Minimum and maximum reinforcement

Minimum and maximum tension reinforcement in a doubly reinforcement beam are same as those in singly reinforced beam. The minimum compression reinforcement can of course be zero.

Design Steps



Step 1: Compute M_{lim}

Determine the limiting moment of resistance M_{lim} for the given cross section using the equation for a singly reinforced beam A.

$$M_{lim} = 0.87\sigma_y A_{t1}(d - 0.42x_m)$$

or

$$M_{lim} = 0.36\sigma_{ck} b x_m (d - 0.42x_m)$$

where A_{t1} is area of tension steel corresponding to a balanced singly reinforced beam.

Step 2: Design of doubly reinforced section

If the factored moment M exceeds M_{lim}, a doubly reinforced section is required to be designed for the additional moment (M - M_{lim}).

$$M - M_{lim} = \sigma_{sc} A_{sc} (d - d') \qquad [\text{ As } \sigma_y >> \sigma_{cc}]$$

where A_{sc} is area of compression steel and σ_{cc} is compression stress in concrete at the level of compression steel.

Step 3: Find additional area of tension steel

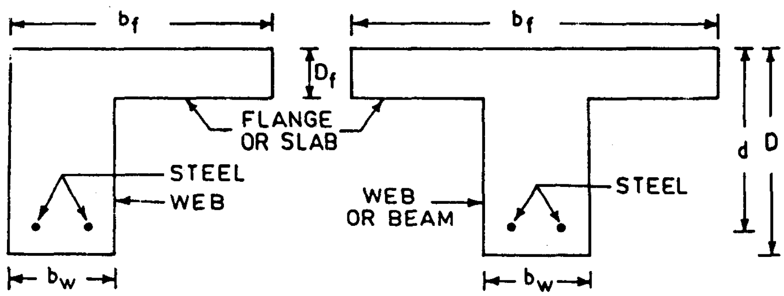
$$A_{t2} = \frac{\sigma_{sc} A_{sc}}{0.87\sigma_y}$$

Step 4: Compute total area of tension steel

$$A_t = A_{t1} + A_{t2}$$

FLANGED (T) BEAMS

In most reinforced concrete structures, concrete slabs and beams are cast monolithic. Thus, beams form part of the floor system together with the slab. In a floor system consisting of several beams cast monolithically with the slab, the intermediate beams act as **T - beams**, and the end beams act as **L - beams**. L and T beams are specified by the following dimensions as shown in given figure.

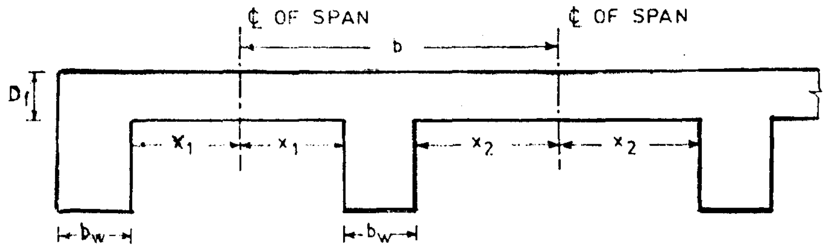


where, b_f = effective width of flange, d = effective depth, b_w = width of web(or rib) or beam and D_f = depth of flange or slab

Effective Width of Flange

Type of beam	Effective flange width(b_f) (lesser will be adopted)	
T beams	$(l_0/6) + b_w + 6D_f$	$x_1 + x_2 + b_w$
Isolated T beams	$\frac{l_0}{(l_0/b) + 4} + b_w$	b
L - beams	$(l_0/12) + b_w + 3D_f$	$x_1 + b_w$
Isolated L beams	$\frac{0.5 l_0}{(l_0/b) + 4} + b_w$	b

where b = actual width of flange, l_0 = distance between points of zero moments in a beam(for continuous beam l_0 = 0.7 x effective span), x_1, x_2 = half the clear distance between two adjacent beams as shown in given figure.



How To Solve Various Problems

Type	Condition	Comp. Force	Tensile Force	Moment Of Resistance
N.A. in flange	Treat beam as Singly/doubly RCC beam.			
Balanced, singly Reinforced, N.A. in web	$D_f/d \leq 0.20$	$0.36\sigma_{ck}b_wx_m+0.446\sigma_{ck}(b_f-b_w)D_f$	$0.87\sigma_ya_t$	$0.36\sigma_{ck}b_wx_m(d-0.42x_m)$ $+0.446\sigma_{ck}(b_f-b_w)$ $+D_f(d-0.5D_f)$
	$D_f/d > 0.20$	$0.36\sigma_{ck}b_wx_m+0.446\sigma_{ck}(b_f-b_w)y_f$	$0.87\sigma_ya_t$	$0.36\sigma_{ck}b_wx_m$
Under reinf. NA in web	$D_f/x \leq 3/7$	$0.36\sigma_{ck}x+0.446\sigma_{ck}(b_f-b_w)D_f$	$0.87\sigma_ya_t$	(i) $0.87\sigma_ya_tz$ (ii)$0.36\sigma_{ck}b_wx(d-0.42x)$ $+0.446\sigma_{ck}(b_f-b_w)D_f.$ $(d-0.5D_f)$
	$D_f/x > 3/7$	$0.36\sigma_{ck}b_wx+0.446\sigma_{ck}(b_f-b_w)y_f$	$0.87\sigma_ya_t$	(i) $0.87\sigma_ya_tz$ (ii)$0.36\sigma_{ck}b_wx(d-0.42x)$ $+0.446\sigma_{ck}(b_f-b_w).$ $Y_f(d-0.5y_f)$
Over reinf., NA in web	$D_f/d \leq 0.20$	$0.36\sigma_{ck}b_wx_m+0.446\sigma_{ck}(b_f-b_w)D_f$	$0.87\sigma_ya_t$	$0.36\sigma_{ck}b_wx_m(d-0.42x_m)$ $+ 0.446\sigma_{ck}(b_f-b_w).$ $D_f(d-0.5D_f)$
	$D_f/d > 0.20$	$0.36\sigma_{ck}b_wx_m+0.446\sigma_{ck}(b_f-b_w)y_f$	$0.87\sigma_ya_t$	$0.36\sigma_{ck}b_wx_m(d-0.42x_m)$ $+0.446\sigma_{ck}(b_f-b_w).$ $Y_f(d-0.5y_f)$

Reinforcement Details

Minimum and maximum reinforcement

- Minimum reinforcement

$$\frac{A_0}{b_wd} = \frac{0.85}{\sigma_y}$$

- Maximum Reinforcement

$$A_{max} = 0.04\, b_w D$$

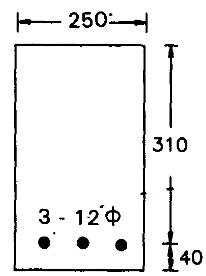
where A_0 is minimum area of tension steel.

Example

Determine the moment of resistance for the section in given figure. Given $\sigma_{ck} = 20\ N/mm^2$, $\sigma_y = 415\ N/mm^2$.

Solution

Given data: Area of steel $A_t = 3x\frac{\pi}{4}x12^2 = 339\,mm^2$, d = 310 mm, b = 250 mm



Step 1: Compute x

$$x = \frac{0.87\sigma_y A_t}{0.36\sigma_{ck} b} = 68\,mm$$

Step 2: Compute x_m

$$x_m = 0.48d = 0.48\,x\,310 = 148.8\,mm$$

Step 3: Decide whether the section is under reinforced or over reinforced

We see that $x < x_m$, Hence the section is under reinforced.

Step 4: Compute lever arm

$$z = d - 0.42x = 310 - 0.42\,x\,68 = 281\,mm$$

Step 5: Compute moment of resistance

$$M_u = 0.87\sigma_y A_t z = 0.87 \text{ x } 415 \text{ x } 339 \text{ x } 281 \text{ N-mm} = 34.40 \text{ kN-m}$$

Example

Design a rectangular beam for an effective span of 6 m. The superimposed load is 80 kN/m and size of the beam is limited to 300 x 700 mm overall. Use M20 mix and Fe 415 grade steel.

Solution

Given data: l = 6 m, b = 300 mm, D = 700 mm, w' = 80 kN/m, self weight w'' = 0.30 x 0.70 x 25 = 5.25 kN/m, $\sigma_{ck} = 20\ N/mm^2$, $\sigma_y = 415\ N/mm^2$.

Let effective cover is 0.1D = 0.1 x 700 = 70 mm. Hence effective depth will be d = 700 - 70 = 630 mm.

Step 1: Compute total load and bending moment

Total load $w = 80 + 5.25 = 82.25$ kN/m

Factored bending moment

$$M = 1.5 \text{ x BM} = 1.5 \text{ x } (wl^2/8) = 1.5 \text{ x } (85.25 \text{ x } 6^2/8) = 576 \text{ kN-m}$$

Step 2: Compute maximum depth of neutral axis x_m

$$x_m = 0.48d = 0.48x630 = 300\,mm$$

Step 3: Compute M_{lim}

$$M_{lim} = 0.36\sigma_{ck} b x_m (d - 0.42x_m)$$

= 0.36 x 20 x 300 x 300 x (630 - 0.42 x 300) = 3.26 x 10^8 N-mm = 326 kN-m

Step 4: Compute A_{t1}

$$A_{t1} = \frac{M_{lim}}{0.87\sigma_y (d - 0.42x_m)}$$

Step 5: Compute strain in compression reinforcement ε_{sc} and stress in compression reinforcement σ_{sc}

$$\varepsilon_{sc} = 0.0035\left(1 - \frac{d'}{x_m}\right) = 0.0035\left(1 - \frac{70}{300}\right) = 2.7\,x\,10^{-3}$$

From table of stresses σ_{sc} in compression steel

∴ σ_{sc} = 353 N/mm^2

Step 6: Compute area of compression steel

$$A_{sc} = \frac{M - M_{lim}}{\sigma_{sc}(d - d')} = \frac{(576 - 326)\,x\,10^6\ N - mm}{353(630 - 70)} = 1265\,mm^2$$

Step 7: Compute additional area of tension steel

$$A_{t2} = \frac{\sigma_{sc} A_{sc}}{0.87\sigma_y} = 1237\,mm^2$$

Step 7: Compute total area of tension steel

$$A_t = A_{t1} + A_{t2} = 1790 + 1237 = 3027\,mm^2$$

Step 8: Provide reinforcement

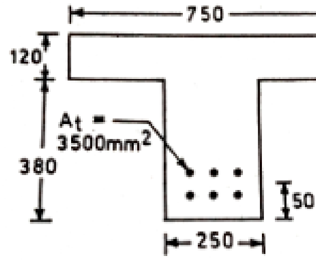
Provide 5 - 28 mm bars at bottom (A_t = 3078 mm^2) and provide 5 - 18 mm bars at top (A_{sc} = 1272 mm^2).

Step 9: Check for maximum tension steel

$$A_{max} = 0.04bD = 0.04\,x\,300\,x\,700 = 8400\,mm^2 > 3078\,mm^2 \text{ Hence O.K.}$$

Example

Calculate the moment of resistance of a T beam as shown in Figure, assuming M15 mix and Fe 415 grade steel.



Solution

Given data : σ_{ck} = 15 N/mm^2, σ_y = 415 N/mm^2, D_f = 120 mm, d = 500 - 50 = 450 mm

b_f = 750 mm, A_t = 3500 mm^2, b_w = 250 mm

Step 1: Make assumption

Let neutral axis (N.A.) lies in web, and section is balanced.

Step 2: Compute D_f/d

$$D_f / d = 120/450 = 0.27 > 0.20$$

$$y_f = 0.15x + 0.65(120) = 0.15x + 78$$

Step 3: Compute x

Force of compression = Force of tension

$$\therefore \quad 0.36\sigma_{ck} b_w x + 0.44\sigma_{ck}(b_f - b_w) y_f = 0.87\sigma_y A_t$$

Solving we getx = 541 mm

Step 4: Find x_m

$$x_m = 0.48 \text{ x } 450 = 216 \text{ mm}$$

Step 5: Decide type of section

As x > x_m, it would be an over reinforced section.

Step 6: Find y_f

$$y_f = 0.15x_m + 0.65(120) = 0.15(216) + 78 = 110 \text{ mm} < D_f \quad \text{O.K.}$$

Step 7: Find moment of resistance

$$M = 0.36\sigma_{ck} b_w x_m (d - 0.42x_m) + 0.44\sigma_{ck}(b_f - b_w) y_f (d - 0.5y_f)$$

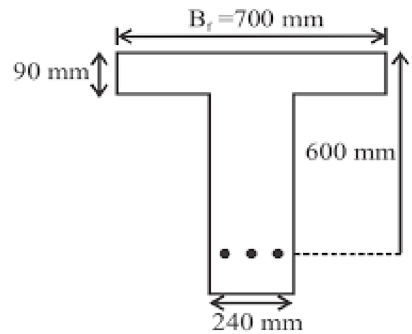
$$= 2.50 \text{ x } 10^8 \text{ N-mm} = 250 \text{ kN-m}$$

Step 8: Find actual moment of resistance

$$M_{ac} = M / \text{load factor} = 250/1.5 = 167 \text{ kNm}$$

Problem (SSC, Junior Engineer, 2015)

Using limit state method (LSM), determine the moment of resistance of the T-Beam as shown in the figure below. Use M 15 concrete and Fe 415 steel.



Answer: 157.01 kNm

Example (SSC, Junior Engineer, 2011)

A room 600 cm long and 500 cm wide has a flat roof. There is one T-beam in the centre (cross section below the slab 30 cm x 50 cm) and the slab is 15 cm thick. Estimate the quantity of iron bars required for reinforcement (for the T beam only) from the data given below

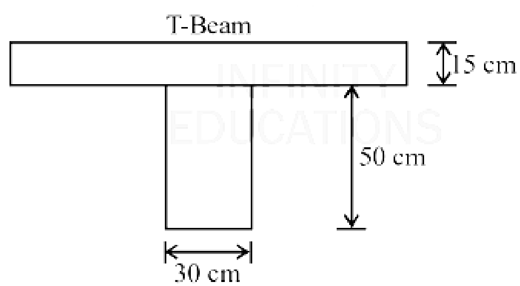
Main bars - *8 nos. of 25 mm dia. in 2 rows of 4 each (all 4 in the bottom being straight and other being bent)*

Stirrups - *10 mm dia. and 15 cm centre to centre throughout.*

Anchor bars - *2 nos of 16 mm dia.*

Solution

See figure.



Room dimensions = 600 cm x 500

Main Bars = 8 (Given)

Stirrups = 15 cm c/c

No. of stirrups = (length/spacing) + 1 = (600/15) + 1 = 41

Anchor bars = 2

Total bars = Main bars + Stirrups + anchor bars = 8 + 41 + 2 = 51

Shear and Bond Stresses

SHEAR STRESS

Shear force is present in beams *where there is a change in bending moment along the span*. It is equal to the rate of change of bending moment.

Clause 40.1 of the code requires that nominal shear stress τ_v be obtained by dividing the factored shear force by the effective area bd, that is,

$$\tau_v = V_u/bd$$

where τ_v = nominal shear stress(horizontal shear stress), V_u = factored shear force at the section

In case of beams of varying depth, the equation is modified as follows:

$$\tau_v = \frac{V_u \pm (M_u/d)\tan\beta}{bd}$$

where, M_u = factored bending moment at the section, β = angle between top and bottom edges of the beam

Shear Reinforcement

When shear stress exceeds the shear capacity of the concrete, shear reinforcement is provided.

To prevent the possibility of crushing of concrete in the web of a member, maximum shear stress values are limited as shown in table.

Maximum Shear Stress

Concrete grade	**M15**	**M20**	**M25**	**M30**	**M35**	**M40**
τ_{cm}**(N/mm²)**	2.5	2.8	3.1	3.5	3.7	4.0

The shear strength of concrete τ_c based on the percentage of longitudinal tensile reinforcement is shown in following table

Design Shear Strength of Concrete(τ_c) N/mm²

100A_s/bd	**M15**	**M20**	**M25**	**M30**	**M35**	**M40**
0.25	0.35	0.36	0.36	0.37	0.37	0.38
0.50	0.46	0.48	0.49	0.50	0.50	0.51
0.75	0.54	0.56	0.57	0.59	0.59	0.60
1.00	0.60	0.62	0.64	0.66	0.67	0.68
1.25	0.64	0.67	0.70	0.71	0.73	0.74
1.50	0.68	0.72	0.74	0.76	0.78	0.79
1.75	0.71	0.75	0.78	0.80	0.82	0.84
2.00	0.71	0.79	0.82	0.84	0.86	0.88
2.25	0.71	0.81	0.85	0.88	0.90	0.92
2.50	0.71	0.82	0.88	0.91	0.93	0.95

Note : The term A_s is the area of longitudinal tension reinforcement which continues at least one effective depth beyond the section being considered.

Thus when shear reinforcement is necessary, the shear strength of the beam is calculated on the following basis:

Total shear strength = Shear resistance of effective concrete area as a function of longitudinal main steel bars plus shear resistance of vertical shear stirrups plus shear resistance of inclined shear stirrups.

Tests have shown that inclined bars alone do not provide a satisfactory solution and their contribution is limited to 50% of the net shear after deducting the contribution of the concrete. The remaining shear resistance is provided by vertical stirrups.

A concrete section can be classified into any one of the following four cases depending upon the amount of factored shear force on the section relative to its shear capacity.

Case 1 : No Shear Reinforcement

The code requires the shear reinforcement need not be provided in the following cases :

(a) where shear force V_u is less than 0.5 times the shear capacity of the section, and

(b) in members of minor structural importance such as lintels.

Case 2 : Minimum shear reinforcement ($\tau_v \le \tau_c$)

Minimum shear reinforcement should be provided if the nominal shear stress τ_v is less than or equal to shear strength of the concrete. The spacing x of the shear stirrups is given by

$$x = \frac{\sigma_y A_0}{0.4b}$$

where A_0 = total cross sectional area of stirrup legs effective in shear, b = breadth of the beam or breadth of the web of flanged beam, and σ_y = characteristic strength of the stirrup reinforcement which should not be taken greater than 415 N/mm^2.

Case 3 : Shear Reinforcement ($\tau_v > \tau_c$)

Adequate shear reinforcement should be provided if the nominal shear stress τ_v exceeds the shear strength of concrete.

Case 4: Redesign of Section ($\tau_v > \tau_{cm}$)

The nominal shear stress due to factored loads should not exceed the maximum permissible shear stress in the concrete. In case the nominal shear stress exceeds this value, the section should be redesigned.

Spacing of shear reinforcement

If we provide *vertical stirrups* as shear reinforcement then spacing of stirrups (x) is given by formula

$$x = \frac{0.87\sigma_y A_{sv} d}{V_{us}} = \frac{0.87\sigma_y A_{sv} d}{V_u - \tau_c bd}$$

where A_{sv} is cross sectional area of stirrups [for two legged stirrups $A_{sv} = 2 \times (\pi/4)d^2$], and V_{us} = net design shear

Maximum spacing of vertical shear stirrups should not exceed 0.75d or 300 mm.

Minimum spacing of vertical shear stirrups should be 100 mm to *permit space for compaction of the concrete.*

BOND STRESS

Bond stress is defined as the shear force per unit of nominal surface area of a reinforcing bar acting parallel to the bar on the interface between the bar and the surrounding concrete. A basic requirement in reinforced concrete structures is that the steel and surrounding concrete act together and there should be no *slip* of the bar relative to its surrounding concrete. In simple beams, *critical* sections generally occur at the face of a support, at a section of maximum stress, and at points where bars may be curtailed theoretically. In continuous beams, points of contraflexure provide additional critical sections.

Development Length

The force in any reinforcing bar must be transmitted to the surrounding concrete by bond, in the embedment length, before the bar may be terminated. This embedment length is called *development length*.

Development length is given by formula

$$L_d = \frac{0.87\sigma_y\phi}{4\tau_{bd}}$$

Design bond stresses for plain steel bars in tension are given below

Conc.	**M15**	**M20**	**M25**	**M30**	**M35**	**M40**
τ_{bd} **N/mm^2**	1.0	1.2	1.4	1.5	1.7	1.9

The code requires that τ_{bd} values may be increased by 60% for deformed steel bars in tension. These values may be further increased by 25% for bars in compression.

If the design bond stress τ_{bd} is not to be exceeded, the following condition must be fulfilled

$$L_d \le \frac{M_1}{V} + L_0$$ [Clause 26.2.3.3(c) of the code]

Also according to code, development length at a simple support is

$$L_d \le 1.3\frac{M_1}{V} + L_0$$

where M_1 is moment of resistance with respect to tension steel alone at section under consideration (generally critical section), V is factored shear force at same section, L_d is development length and L_0 is sum of anchorage beyond the centre of support and equivalent *anchorage value* of any hook/bend. L_0 is limited to the effective depth of the member or 12ϕ *whichever is greater* at a point of inflection.

Clause 26.2.2.1 of the code gives the anchorage values of bends and hooks as follows:

- The anchorage value of a bend should be taken as 4ϕ for each 45^0 bend subject to a maximum of 16ϕ.
- The anchorage value of 90^0 bend is 8ϕ.
- The anchorage value of a standard U type hook is 16ϕ.

Reinforcement Splicing

When two reinforcing bars are to be *joined* to make a *longer* one, it is necessary to overlap a length sufficient to develop its full strength by bond round the surface so that it does not slip under the design stress. The following general rules should be followed for lap splicing :

- Lap splices should not be used for bars larger than 36 mm. Larger diameter bars may be welded together.

- Lap length including anchorage value of hooks in *flexural tension* is L_d or 30ϕ, whichever is greater
- Lap length including anchorage value of hooks in *direct tension* is $2L_d$ or 30ϕ whichever is greater.
- The straight length of the lap should not be less than 200 mm or 15ϕ.
- The lap length is calculated on the basis of diameter of smaller bar when bars of two different diameter are to be spliced.

Example

An RC beam has an effective depth of 500 mm and a breadth of 350 mm. It contains 4-25 mm bars. If σ_{ck} = 15 N/mm^2 and σ_{sv} = 250 N/mm^2, calculate the shear reinforcement needed for factored shear force of 350 kN.

Solution

Step 1: Compute percentage area of longitudinal steel

$$p = \frac{100A_s}{bd} = \frac{100x4x\pi/4x25^2}{350x500} = 1.12\%$$

Step 2: Compute design shear strength of concrete

From table of design shear strength of concrete,

τ_c = 0.62 N/mm^2 (for M15 concrete)

Step 3: Compute nominal shear stress

$$\tau_v = \frac{V_u}{bd} = \frac{350x1000}{350x500} = 2\,N/mm^2$$

Step 4: Compute maximum shear stress in concrete

From table of maximum shear stress of concrete

τ_{cm} = 2.5 N/mm^2 (For M15 concrete)

Step 5: Decision about shear reinforcement

We see that 0.62 N/mm^2 < 2 N/mm^2 < 2.5 N/mm^2, therefore provide shear reinforcement.

Step 6: Spacing of shear reinforcement

$$x = \frac{0.87\sigma_y A_{sv} d}{V_{us}} = \frac{0.87\sigma_y A_{sv} d}{V_u - \tau_c bd}$$

Let us use 8 mm - 2 legged vertical stirrups.

Now $$A_{sv} = 2x\frac{\pi}{4}x8^2 = 100.5\,mm^2$$

and $$V_{us} = V_u - \tau_c bd = (350x1000) - 0.62x350x500 = 241500\,N$$

$\therefore$ Spacing of 8 mm - 2 legged vertical stirrups

$$x = \frac{0.87x250x100.5x500}{241500} = 46\,mm$$

Step 7: Check for maximum spacing

As x = 46 mm < 300 mm and 0.75d = 0.75 x 500 = 375 mm, hence O.K.

Step 8: Check for minimum reinforcement

$$A_0 \geq \frac{0.4bx}{0.87\sigma_y} = \frac{0.4x350x45.25}{0.87x250} = 30\,mm^2 < A_{sv} \text{ O.K.}$$

Step 9: Check for minimum spacing

We see that x = 46 mm < 100 mm Hence not O.K.

Step 10: Compute revised area of stirrups

Let us keep minimum spacing of stirrups equal to 100 mm in order to permit space for compaction of the concrete.

Hence revised area of stirrups will be

$$A_{sv} = \frac{V_{us}x}{0.87\sigma_y d} = \frac{241500x100}{0.87x250x500} = 222\,mm^2$$

Let us use 12 mm - 2 legged vertical stirrups @ 100 mm c/c.

Step 9: Check for minimum reinforcement for revised stirrups

$$A_0 \geq \frac{0.4bx}{0.87\sigma_y} = \frac{0.4x350x100}{0.87x250} = 65\,mm^2 < A_{sv} \text{ of revised stirrups. Hence O.K.}$$

Step 10: Check for maximum spacing for revised stirrups

As x = 100 mm < 300 mm and < 0.75d = 0.75 x 500 = 375 mm, hence O.K.

Example

A simply supported beam is 25 cm x 50 cm and has 2-20 mm tor bars going into support. If the shear force at the centre of support is 110 kN at working loads, determine the anchorage length. Assume M20 mix and Fe415 grade steel.

Solution

Step 1: Compute factored shear force

$$V = 1.5 \text{ x } 110 = 165 \text{ kN}$$

Step 2: Find effective depth

Let clear cover is 25 mm then effective depth will be

$$d = 500 - 25 - (20/2) = 465 \text{ mm}$$

Step 3: Find depth of neutral axis x and moment of resistance M_1

$$x = \frac{0.87\sigma_y A_t}{0.36\sigma_{ck} b} = 126\,mm < x_m \qquad \text{Hence O.K.}$$

$$M_1 = 0.87\sigma_y A_t (d - 0.42x) = 93.45 \text{ x } 10^6 \text{ N} - \text{mm}$$

Step 4: Find bond stress for M20 mix

From table of design bond stress

τ_{bd} = 1.2 N/mm^2

Now increase it by 60% for tor steel.

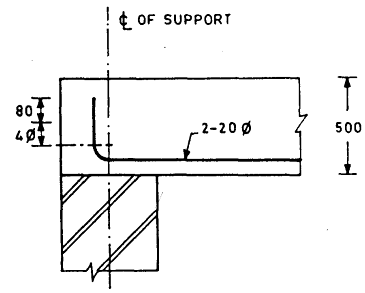
∴ τ_{cd} = 1.6 x 1.2 N/mm^2

Step 5: Find development length

$$L_d = \frac{0.87\sigma_y\phi}{4\tau_{bd}} = \frac{0.87(415)\phi}{4(1.6x1.2)} = 47\phi$$

Step 6: Provide bend

let us we provide 90^0 bend at centre of support. Its anchorage value is 8ϕ i.e. 8 x 20 = 160 mm. Hence L_0 = 160 mm.



Step 7: Check for anchorage value

$$L_d \le 1.3\frac{M_1}{V} + L_0$$

i.e. $$47\phi \le \frac{1.3x93.45x10^6}{165x1000} + 160 \Rightarrow \phi \le 19\,\text{mm}$$

Now diameter provided is 20 mm. NOT O.K.

Hence increase the anchorage length. Let us increase L_0 to 240 from 160 mm (See figure)

Now we see that by applying above check once again

$\phi \le 20.8$ mm Now O.K.

Note: In step 6, instead of 90^0 bend if we provide U-bend at centre of support then L_0 = 16ϕ = 320 mm.

After applying a check for anchorage value, we will see that $\phi \le 22.47$ mm, which is perfectly O.K. because diameter provided is 20 mm which is less than 22.47 mm.

Example (SSC, Junior Engineer, 2013)

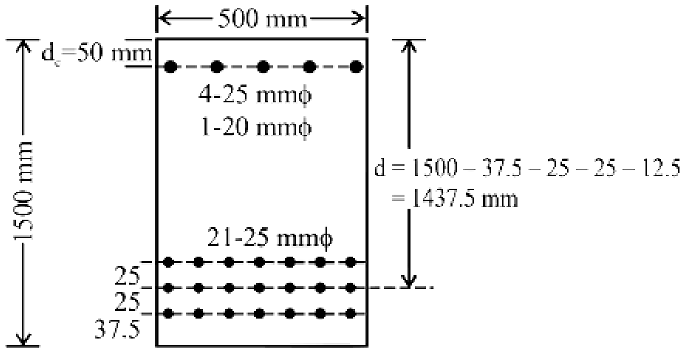
A simply supported 18 m effective span RCC rectangular beam of 500 mm x 1500 mm (over all depth) section is reinforced throughout with 21 nos. 25 mm diameter bars in three layers of 7 bars each at a clear cover of 37.5 mm on tensile face. The reinforcement on the compression face is 4 - 25 mm + 1 - 20 mm diameter bars in one layer at an effective cover of 50 mm. The clear cover between the different layers on tension face is 25 mm. M 25 grade concrete and Fe 415 grade steel bars are used in the beam throughout. The beam is laterally restrained throughout the span. (a) What shall be the superimposed uniformly distributed load w, that the beam can carry at working conditions? (b) Design the shear reinforcement at support if design shear strength of concrete τ_c is given as follows for different values of p = 100 A_s/bd.

p	*1.25*	*1.5*	*1.75*
τ_c(MPa)	*0.70*	*0.74*	*0.78*

Solution

Part (a)

See figure.



f_y = 415 N/mm^2; X_u, lim = 0.48d = 0.48 x 1400 = 672 mm

f_{ck} = 25 N/mm^2; d = 1437.5 mm; d' = 1500 – 37.5 – 25 – 25 – 12.5 = 1400 mm

d_c = 50 mm; $f_{sc} = \sigma_{sc}$ = 0.9566f_y = 0.9566 x 415 = 396.989 = 397 N/mm^2

$$A_{sc} = 4\left(\frac{\pi}{4}\right)25^2 + 1\left(\frac{\pi}{4}\right)20^2 = 2277.65\,mm^2$$

$$MR = 0.36 f_{ck} B x_{u\lim}(d - 0.42x_{u\lim}) + (f_{sc} - 0.45 f_{ck})A_{sc}(d' - d_c)$$

= 0.36 x 25 x 500 x 672(1437.5 – 0.42 x 672)

+ (397 – 0.45 x 25) x 2277.65 x (1400 – 50)

= 4679.62 kNm

$$M_{u\lim} = \frac{w_u l^2}{8} \Rightarrow w_u = \frac{4679.62x8}{18x18} \Rightarrow w_u = 115.54\,kN/m$$

$$w_{total} = \frac{w_u}{1.5} = \frac{115.54}{1.5} = 77.03\,kN/m$$

w_{total} is super imposed UDL along with the self weight of the beam. Hence, the self weight of beam should be deducted.

Self weight of beam = 0.5 x 1.5 x 1 x 25 = 18.75 kN/m

∴ w = w_{total} – 18.75 = 77.03 – 18.75 = 58.28 kN/m

Part (b)

w_u = 115.54 kN/m

V_u = w_ul/2 = (115.54 x 18)/2 = 1039.86 kN

∴ Nominal shear stress

$$\tau_v = \frac{V_u}{Bd} = \frac{1039.86x10^3}{500x1400} = 1.48\,N/mm^2$$

Now $$p = 100x\frac{A_{st}}{Bd} = \frac{100x10308.35}{500x1400} = 1.47\%$$

For p = 1.25%, we have τ_c = 0.70 N/mm^2

∴ $$\tau_c \; for\,1.47\% = 0.70 + \frac{(0.74-0.70)}{(1.50-1.25)}(1.47-1.25) = 0.73\,N/mm^2$$

∴ $V_c = \tau_c Bd = 0.73 \times 500 \times 1400 = 511$ kN

$\tau_v > \tau_c$, hence shear reinforcement is required.

$V_{us} = V_u - V_c = 1039.86 - 511.00 = 528.86$ kN

Now shear reinforcement at supports should be provided according the IS: 456-2000.

$$S_v = \frac{0.87 f_y A_{sv} d}{V_{us}}$$

Hence, providing 200 mm spacing at supports.

Minimum shear reinforcement

$$S_v \le \frac{2.5 A_{sv} f_y}{b} \Rightarrow S_v \le \frac{2.5(157.08)(415)}{500} \Rightarrow S_v \le 325.94\,mm \qquad [\because b = B]$$

Hence, at mid span spacing between stirrups can be 300 mm c/c.

Maximum spacing = 0.75 d = 0.75 x 1400 = 1050 mm

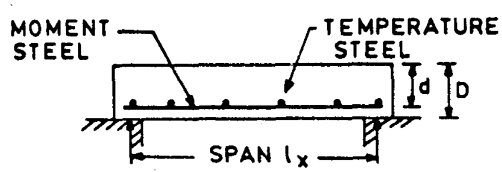
Hence proved reinforcement is OK.

Slabs

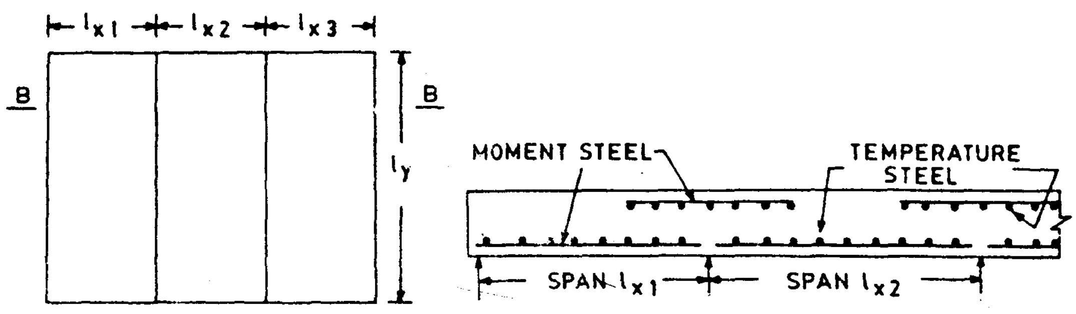
ONE WAY SLABS

One way slabs are those in which the length is more than twice the breadth. A one way slab can be simply supported or continuous.

One way simply supported slab is shown in given figure.



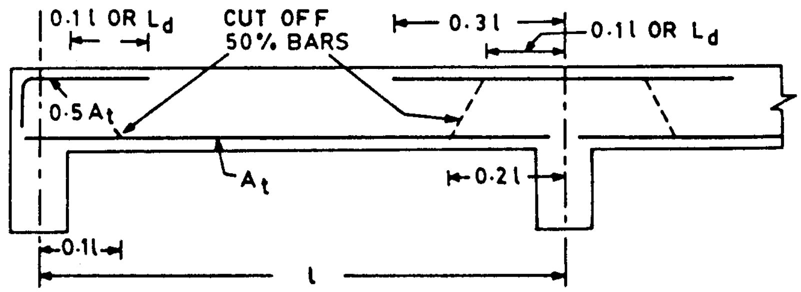
One way continuous slab is shown in following figure.



(a) Plan **(b) Section at B-B**

The effective spans under different boundary conditions are the same as those for beams discussed earlier. A continuous one way slab can be analysed in a manner similar to that used for a continuous beam.

The requirements for continuous slabs of approximately equal spans where the live load does not exceed the dead loads are shown in following figure. As shown in this figure, negative steel is provided in a distance 0.1l or L_d(= 45ϕ for M20 mix) whichever is greater.



Shear stresses in slabs are generally not critical under normal loads but should be checked. Clause 40.2.1.1 of the code permits in solid slabs thinner than 300 mm an increase in shear strength τ_c of concrete by a factor k is given in table.

Value of k for solid slabs

D_s(mm)	≥ 300	275	250	225	200	175	≤ 150
k	1.00	1.05	1.10	1.15	1.20	1.25	1.30

In solid slabs, the nominal shear stress τ_v should be less than $k\tau_c$. Shear reinforcement may be provided in slabs deeper than 200 mm. The development length is checked at the same critical points as for beams.

The check for deflection is a very important consideration in the slab design.

Deflection Control

For a slab to be safe in deflection

$$\frac{L}{d} \leq \alpha\beta\gamma\delta\lambda$$

where L = span; d = effective depth; α = basic values of span to effective depth ratios for spans upto 10 m. It is 7 for cantilever, 20 for simply supported slab and 26 for continuous slab; β = a factor which accounts for correction in the values of α for spans greater than 10 m = 10/span, where span is in meter. β will be equal to unity if span is less than 10 m; γ = a factor which depends on the stress at service and amount of steel for a tension reinforcement.; δ = a factor which depends on the area of compression reinforcement; and λ = a factor for flanged beams which depends on the ratio of web width to the flange width.

Reinforcement Details

Nominal cover

It is same as that for beams. However in slabs having main reinforcement upto 12 mm diameter bars, for mild exposure, the nominal cover may be reduced to 20 mm instead of 25 mm.

Maximum distance between bars in tension

- The horizontal distance between parallel main reinforcement bars should not be more than three times the effective depth of a solid slab or 300 mm whichever is smaller.
- The horizontal distance between parallel reinforcement bars provided against shrinkage and temperature should not be more than five times the effective depth of a solid slab or 450 mm whichever is smaller.

Minimum Reinforcement

The minimum reinforcement in either direction in slabs should not be less than 0.15% of the total cross sectional area when using mild steel reinforcement, and 0.12% of the total cross sectional area when using high yield strength reinforcement. Further, maximum diameter of reinforcing bar should not exceed one eighth of the total thickness of slab.

Curtailment of bars

The general recommendations for curtailment of bars given in clause 26.2.3 of the code apply for slabs also.

TWO WAY SLABS

When slabs are supported on four sides, two way spanning action occurs. Such slabs may be simply supported or continuous on any or all sides. The design of two way slabs is based on tables of bending moments and design specifications given in clause 24.4 of the code.

Simply Supported Two Way Slabs

The values of bending moments used for the design of such slabs can be obtained as follows:

$$M_x = \alpha_x w l_x^2 \text{ and } M_y = \alpha_y w l_y^2$$

where M_x, M_y are maximum moments at mid span on strips of unit width and spans l_x and l_y, respectively. l_x is length of *shorter* side, l_y is length of *longer* side and α_x, α_y are moment coefficients given in following table.

l_y/l_x	1.0	1.1	1.2	1.3	1.4	1.5	1.75	2.0	2.5	3.0
α_x	0.062	0.074	0.084	0.093	0.099	0.104	0.113	0.118	0.122	0.124
α_y	0.062	0.061	0.059	0.055	0.051	0.046	0.037	0.029	0.020	0.014

The code requires that at least 50% of the tension reinforcement provided at midspan should *extend* to within $0.1l_x$ or $0.1l_y$ of the support, as appropriate.

Restrained Slabs

A slab may have its few or all edges restrained. A hogging or negative bending moment will develop in the top face of the slab at the supported sides. In these slabs, the corners are prevented from lifting and provision is made for tension.

For these slabs

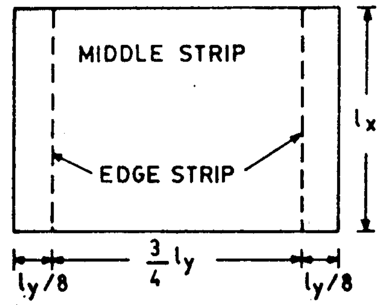
$$M_x = \beta_x w l_x^2 \text{ and } M_y = \beta_y w l_y^2$$

where β_x and β_y are moment coefficients given in BS 8110-part 1 - 1985.

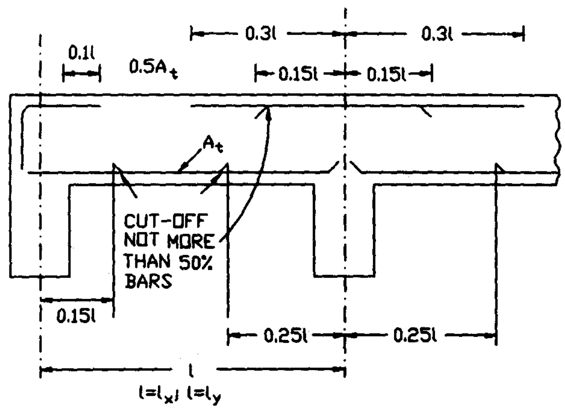
Design Rules

Design rules for two way slabs are given in appendix C of the code.

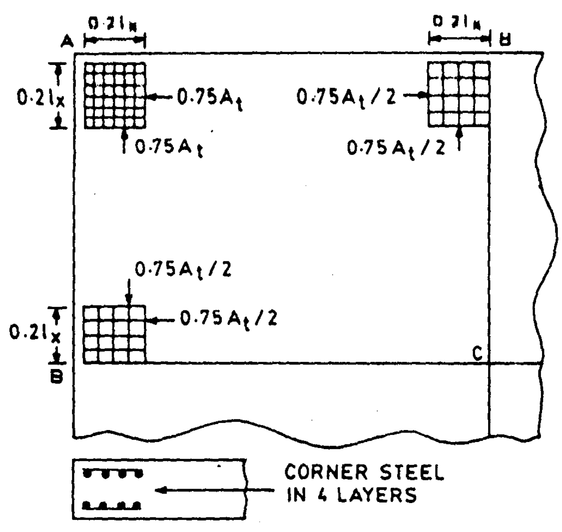
- The maximum positive and negative moments apply only to the middle strips subject to the requirements for minimum areas of reinforcement. The bars are uniformly spaced in the middle strips.



- The rules for curtailment of bars are given below in the figure.

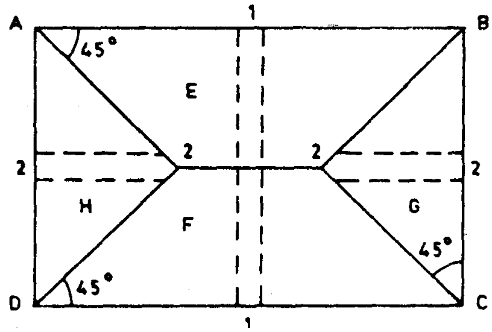


- Negative moments arise at discontinuous edges. Tension reinforcement equal to 50% of that at the mid span is provided. This steel should extended 0.1 of the span into the span as shown above.
- Torsion or corner reinforcement must be provided at corners A and B of the slabs as shown in figure.



Design for shear

The slab thickness must be sufficient for resisting the shear. The shear requirement may be investigated by observing strips 1-1 and 2-2 of unit width as shown in figure.



Loads on E and F will go to long beams and loads on triangular portions will go to shorter beams.

Shear force in both strips have the following value

$$V_u = w_u L_x / 2$$

In two way slabs, the short span bars are provided in the bottom layer and long span bars are provided in the upper layer.

The nominal shear stress acting on slab is given by

$$\tau_v = V_u / bd$$

The value of τ_v must be less than $k\tau_c$.

Development length should be checked as usual.

Check for deflection

Two way slabs should be checked for deflection to satisfy the serviceability requirement. The strip of slab may be checked against shorter span to effective depth ratio as already discussed. However for two way slabs having both spans upto 3.5 m and a maximum live load of 3000 N/m^2, clause 24.1 of the code permits that short span to overall depth ratios, given in table below, may be assumed to satisfy the vertical deflection limits.

Short span to overall depth ratio

Type of slab	Type of reinforcement	
	Mild steel	Fe 415
Simply supported	35	28
Continuous	40	32

Example

Design a simply supported roof slab for a room 8m x 3.5 m clear in size if the superimposed load is 5 kN/m^2. Use M15 mix and Fe 415 grade steel.

Solution

Step 1: Compute max. bending moment and max. shear force

Here we see that length > 2 x width, hence one way slab.

Consider a 100 cm wide strip of the slab parallel to its shorter span.

Minimum depth of slab

$$d = \frac{L}{\alpha\beta\gamma\delta\lambda}$$

Let $\alpha = 20$, $\beta = 1$, $\gamma = 1$, $\delta = 1$ and $\lambda = 1$

$\therefore$ d = 3500/20 = 175 mm

Let us adopt overall depth D = 190 mm

Dead load of slab = 0.19 x 1.0 x 25 = 4.75 kN/m

Superimposed load = 5 x 1 = 5 kN/m

Total load = 9.75 kN/m

Factored load if the load factor is 1.5 = 1.5 x 9.75 = 14.63 kN/m

Assume steel consists of 10 mm bars with 15 mm clear cover.

Effective depth = 190 - 15 - 5 = 170 mm

Effective span of slab = 3.5 + d = 3.5 + 0.17 = 3.67 m

$\therefore$ Max. bending moment

$$M = w_u l^2/8 = 14.63 \times (3.67^2/8) = 24.63 \text{ kN-m}$$

Max. shear force

$$V = w_u l_c/2 = 14.63 \times (3.5/2) = 25.60 \text{ kN}$$

Step 2: Compute area of tension steel

Depth of the slab is given by

$$M = 0.138\sigma_{ck} b d^2$$

or d = 109 mm (After putting all values)

Adopt effective depth d = 150 mm and D = 170 mm

Area of tension steel is given by

$$M = 0.87\sigma_y A_t \left\{ d - \frac{\sigma_y A_t}{\sigma_{ck} b} \right\}$$

or $A_t = 510$ mm^2 (After putting all values)

Use 10 mm bars @ 150 mm c/c giving total area = 523 mm^2 > 510 mm^2 O.K.

Bend alternate bars at L/7 from face of support where moment reduces to less than half its maximum value.

Step 3: Compute area of temperature reinforcement

Temperature reinforcement equal to 0.15% of the gross concrete area will be provided in the longitudinal direction which will be equal to 0.0015 x 1000 x 170 = 255 mm^2. Use 6 mm MS bars @ mm c/c giving total area = 28 x (1000/100) = 280 mm^2 > 255 mm^2 O.K.

Step 4: Check for shear

$$\text{Percent tension steel} = \frac{100A_t}{bd} = \frac{100(78.5 \times 1000/300)}{1000 \times 150} = 0.17\ \%$$

Shear strength of concrete for 0.17% steel, $\tau_c = 0.35$ N/mm^2

For 170 mm thick slab, k = 1.25

$\therefore$ $\tau_c' = k\tau_c = 1.25 \times 0.35 = 0.44$ N/mm^2

Nominal shear stress $\tau_v = V_u/bd = 25600/1000\text{x}150 = 0.17$ N/mm$^2 < \tau_c'$ O.K.

Hence the slab is safe in shear.

Step 5: Check for development length

Moment of resistance offered by 10 mm bars @ 300 mm c/c.

$$M_1 = 0.87\sigma_y A_t\left\{d - \frac{\sigma_y A_t}{\sigma_{ck} b}\right\} = 13.48 \times 10^6 \text{ N mm}$$

Given V = 25600 N

Let us assume anchorage length $L_0 = 0$

$$L_d \leq 1.3\frac{M_1}{V}$$

$$\therefore \quad 56\phi \leq 1.3\frac{13.48 \times 10^6}{25600} \text{ i.e. } \phi < 12 \text{ mm} \quad \text{O.K.}$$

The code requires that the bars must be carried into the supports by at least $L_d/3 = 190$ mm.

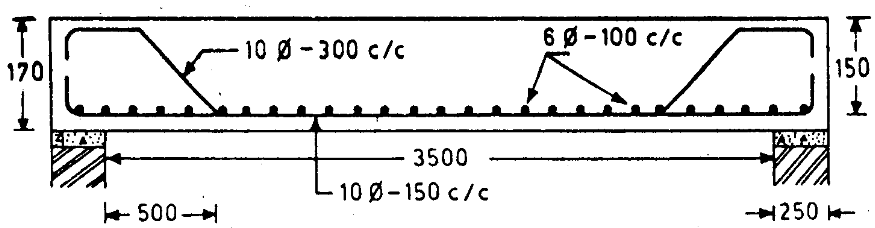
Step 6: Check for deflection

$$\text{Percent tension steel at midspan} = \frac{100A_s}{bd} = \frac{100 \ 78.5 \times 1000/150}{1000 \times 150} = 0.35\ \%$$

$\therefore \gamma = 1.35$ (From code 456:2000), $\beta = 1$, $\delta = 1$, and $\lambda = 1$

Allowable L/d = 20 x 1.35 = 27

Actual L/d = 3670/150 = 24.5 < 27 O.K.



The details of reinforcement are shown in given figure.

Problem (SSC, Junior Engineer, 2017)

A classroom is of the size 8.5 m x 3.6 m. Design a simply supported roof slab for this room. The superimposed load is 5 kN/m. Use M 20 grade concrete and HYSD Fe415 steel. Use limit state method.

$100A_{st}/bd$	*0.15*	*0.25*	*0.50*	*0.75*	*1.0*
τ_c (N/mm^2)	*0.19*	*0.36*	*0.49*	*0.57*	*0.64*

Columns

A column may be defined as an element used primarily to support axial compressive loads and with a height of at least three times its lateral dimension. A column may be classified as short or long column depending on its effective slenderness ratio. The ratio of effective column length to least lateral dimension is referred to as effective slenderness ratio. A short column has a maximum slenderness ratio of 12 and a long column has a slenderness ratio greater than 12. Maximum slenderness ratio of column should not exceed 60.

EFFECTIVE LENGTH

The effective length of a column is defined as the length between the points of contraflexure of the buckled column. The code has given certain values of the effective length for normal usage assuming idealized end conditions shown in given table.

Effective length of compression members

Degree of end restraint of the member	**Effective length**
Effectively held in position and restraint against rotation at both ends	0.65L
Effectively held in position at both ends, restrained against rotation at one end	0.80L
Effectively held in position at both ends but not against rotation	L
Effectively held in position and restrained against rotation at one end, and at the other restrained against rotation but not held in position.	1.2L
The other partially restrained against rotation but not held in position	1.5L
The other end not held in position nor restrained against rotation	2.0L

SHORT COLUMN UNDER AXIAL COMPRESSION

When minimum eccentricity does not exceed 0.05 times D the lateral dimension, clause 39.3 of the code permits the design of short axially loaded compression members by the following equation

$$P_u = 0.4\sigma_{ck}A_c + 0.67\sigma_y A_{sc} = 0.4\sigma_{ck}\left\{A_g - \frac{pA_g}{100}\right\} + 0.67\sigma_y \frac{pA_g}{100}$$

where P_u = Factored axial load, A_c = Area of concrete, may be taken equal to gross area and A_{sc} = Total area of reinforcement

Above equation can be re-written as

$$\frac{P_u}{\sigma_{ck}bD} = 0.4 + \frac{p}{100\sigma_{ck}}(0.667\sigma_y - 0.4\sigma_{ck})$$

where A_g = gross area of cross section = bD and p = percentage of reinforcement

REINFORCEMENT

Longitudinal Reinforcement

- The minimum area of cross section of longitudinal bars must be at least 0.8 % of the gross sectional area of the column.
- The maximum area of cross section of longitudinal bars must not exceed 6 % of the gross cross sectional area of concrete.
- The bars should not be less than 12 mm in diameter.
- The minimum number of longitudinal bars provided in a column must be four in rectangular columns and six in circular columns.
- A reinforced concrete column having helical reinforcement must have at least six bars of longitudinal reinforcement within the helical reinforcement.
- Spacing of longitudinal bars measured along the periphery of a column should not exceed 300 mm.
- Nominal cover for a longitudinal reinforcing bar should not less than 40 mm, nor less than the diameter of such bar. Such a large cover is required so as to prevent buckling of the main longitudinal bars under compression.

  In case of columns of minimum dimensions of 200 mm or under, whose reinforcing bars do not exceed 12 mm a cover of 25 mm may be used.

Transverse Reinforcement

Transverse reinforcement may be in the form of lateral ties or spirals.

Laterals Ties

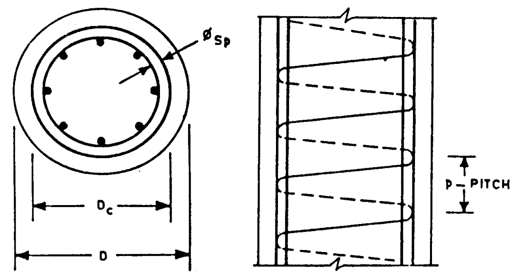
- The diameter of lateral ties should not be less than one fourth of the diameter of the largest longitudinal bar, and in no case less than 6 mm.
- The pitch of the lateral ties should not exceed (a) The least lateral dimension of the compression member (b) 16 times the smallest diameter of the longitudinal reinforcement bar to be tied, and (c) 300 mm.
- Nominal cover to reinforcement is same as that for beams.

Helical Reinforcement

- The diameter of the helical reinforcement should not be less than one fourth of the diameter of the largest longitudinal bar, and in no case less than 6 mm.
- Its pitch should not exceed (if an increased load on column is allowed): (a) 75 mm (b) one sixth of the core diameter of the column
- The pitch should not be less than (a) 25 mm (b) 3 times the diameter of the steel bar forming the helix.
- If an increased load on the column on the strength of helical reinforcement is *not* allowed for, its pitch should not exceed (a) the least lateral dimension of the compression member (b) 16 times the smallest diameter of the longitudinal bar to be tied, and (c) 300 mm.
- Nominal cover to reinforcement is same as that for beams.

COLUMN WITH HELICAL REINFORCEMENT

Please refer to given figure.



If ρ_s be the ratio of volume of spiral reinforcement to the total volume of core (out to out of spirals)

Then $\rho_s = \dfrac{A_{sp}}{A_c}$

Now $\rho_s \geq 0.36\left\{\dfrac{A_g}{A_c} - 1\right\}\dfrac{\sigma_{ck}}{\sigma_{sp}}$

and also $\rho_s = \dfrac{\text{Volume of spiral in one loop}}{\text{Volume of core for a length(p)}} = \dfrac{4a_{sp}}{D_c p}$

where σ_{sp} = characteristic strength of the helical reinforcement bars not exceeding 415 N/mm^2, p = pitch of helix, A_c = area of core = $(\pi/4)D_c$, D_c = diameter of core, outer to outer of spirals, ϕ_{sp} = diameter of spiral wire, a_{sp} = area of spiral wire = $(\pi/4)\phi_{sp}$

$$a_{sp} = 0.09pD_c\left\{\frac{A_g}{A_c} - 1\right\}\frac{\sigma_{ck}}{\sigma_{sp}}$$

Clause 39.4 of the code permits a 5% increase in the strength of a spiral column over that of a similar column with lateral ties.

Example (SSC, Junior Engineer, 2016)

Design a circular column to carry an axial load of 1500 kN using (i) lateral ties (ii) helical reinforcement. Use M25 mix and Fe 415 grade steel.

Solution

(i) Using lateral ties

Step 1: Compute factored load

The factored load is 1.5 x 1500 = 2250 kN

Step 2: Find gross area of section A_g

We know $P_u = 0.4\sigma_{ck}A_c + 0.67\sigma_y A_{sc}$

Assuming 0.8% steel

$$2250x1000 = 0.4x25(A_g - 0.008A_g) + 0.67x415x0.008A_g$$

Solving A_g = 185270 mm^2

Step 3: Decide diameter of column

$$A_g = \frac{\pi}{4}D^2 \Rightarrow 185270 = (\pi/4)D^2 \Rightarrow D = 486\,\text{mm}$$

Hence adopt overall diameter as 500 mm.

Step 4: Provide longitudinal reinforcement

Area of longitudinal steel will be 0.8% of total area = 0.008 x 185270 = 1482 mm^2

Use 8 - 16 mm longitudinal bars.

Step 5: Provide lateral ties and check for pitch (p)

Use 6 mm MS lateral ties.

For pitch $p \le 500$ mm (lateral dimension) and also $\le 16\phi_L$ (16 x 16 = 256 mm) and ≤ 300 mm.

Hence adopt a pitch of 250 mm c/c.

(ii) Using helical reinforcement

Step 1: Find designed factored load

As strength of a column with helical reinforcement is 1.05 times the strength of a similar member with lateral ties.

$\therefore$ Design factored load = (1500 x 1.5/1.05) = 2142.85 kN

Step 2: Find gross area of section A_g

$$P_u = 0.4\sigma_{ck}A_c + 0.67\,\sigma_y A_{sc}$$

Assuming 0.8 % steel

$$2142.85 \times 1000 = 0.4 \times 25(A_g - 0.008A_g) + 0.67 \times 415 \times 0.008A_g$$

Or $A_g = 176\,460$

Step 3: Decide diameter of column

$$A_g = \frac{\pi}{4}D^2 \Rightarrow 176460 = (\pi/4)D^2 \Rightarrow D = 474\,\text{mm}$$

Adopt an overall diameter of 470 mm.

Step 4: Provide longitudinal reinforcement

Area of longitudinal steel = 0.008 x 176 460 = 1412 mm^2

Use 10 - 14 mm bars giving 1538 mm^2 area at a clear cover of 40 mm.

Step 5: Provide helix and check for pitch

Assume 6 mm MS bars for helix. The code requires that :

$$\rho_s \ge 0.36\left\{\frac{A_g}{A_c} - 1\right\}\frac{\sigma_{ck}}{\sigma_{sp}}$$

$$D_c = 470 - 2(40 - 6) = 402 \text{ mm}$$

$$A_g/A_c = 470^2/402^2 = 1.37$$

$$\rho_s \ge 0.36\{1.37 - 1\}\frac{25}{250} \text{ i.e. } \rho_s \ge 0.0133$$

also $\rho_s = \dfrac{4a_{sp}}{D_c p}$ where $a_{sp} = (\pi/4)(6)^2 = 28.26$ mm^2

$$\therefore \quad \rho_s = \frac{4 \times 28.26}{402 \times p}$$

Now equating both values of ρ_s.

$$\frac{4 \times 28.26}{402p} > 0.133 \text{ i.e. } p < 21 \text{ mm}$$

The code requires that the pitch

$$p < 75 \text{ mm and } < D_c/6 \ (402/6 = 67 \text{ mm}) \text{ and} > 25 \text{ mm and} > 3\ \phi_{sp}\ (= 18 \text{ mm})$$

Adopt 8 mm helix instead of 6 mm @ 25 mm c/c.

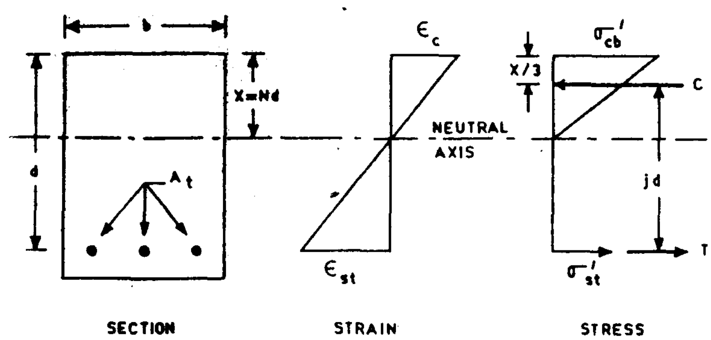
Problem

Design a circular column with helical reinforcement subjected to a working load of 1500 k.N. Diameter of the column is 450mm. The column has unsupported length of 3.5m and is effectively held in position at both ends but not restrained against rotation. Use limit state designing method. Use M-25 concrete and HYSD Fe-415 steel.

Working Stress Method

SINGLY REINFORCED BEAM

Stress-strain relation for a rectangular section in accordance with the working stress method is shown in following figure.



$$N_b d = \frac{d}{1 + (\sigma_{st}' / m\sigma_{cb}')}$$

where N_b is coefficient of balanced depth, d is effective depth of the section, σ_{st}' is computed tensile stress in tension steel, σ_{cb}' is computed compressive stress in the extreme fibre of concrete, m is modular ratio = $280/3\sigma_{cb}$, σ_{cb} is permissible compressive stress in bending in concrete.

The depth of neutral axis is determined by taking the moment of effective areas about the neutral axis. Concrete in tension is assumed to have cracked.

$$b(x^2/2) - mA_t(d - x) = 0$$

where b is breadth of section, x = Nd = depth of neutral axis, N is coefficient of depth of neutral axis, and A_t is total area of tension steel.

Force of compression in concrete

$$C = bNd\frac{\sigma_{cb}'}{2}$$

Force of tension in steel

$$T = A_t\sigma_{st}'$$

Lever arm $jd = z = d - Nd/3$

Moment of resistance with respect to compression

$$= 0.5bNd\sigma_{cb}'(d - Nd/3)$$

Moment of resistance with respect to compression

$$= A_t\sigma_{st}'(d - Nd/3)$$

Permissible stresses in concrete are given below.

Grade of concrete	Compression		Av. bond stress for plain bars in tension τ_{bd} N/mm^2
	Bending σ_{cb}, N/mm^2	**Direct σ_c, N/mm^2**	
M15	5	4	0.6
M20	7	5	0.8

Permissible tensile stress for mild steel upto and including 20 mm diameter is 140 N/mm^2 and that for over 20 mm diameter is 130 N/mm^2. For compression steel in column, it is 130 N/mm^2.

The shear provisions in working stress method are similar to those in limit state design method. Working shear force is used instead of factored shear force. Permissible shear stress values τ_c and maximum permissible shear stress values τ_{cm} for working stress method are given in IS code.

The shear reinforcement can be provided to carry a shear equal to $V_s = (V - \tau_c bd)$. Spacing of vertical stirrups is given by $x = \sigma_{sv}A_{sv}d/V_s$.

Development length of reinforcement can be obtained by using same equation as that for limit state design method.

Example (SSC, Junior Engineer, 2014, Working stress method)

A rectangular, singly reinforced beam 300 mm wide and 500 mm effective depth is used as a simply-supported beam over an effective span of 6 m. The reinforcement consists of 4 bars of 20 mm dia. If the beam carries a load of 12 kN/m (inclusive of self weight), determine the stress developed in concrete and steel. Take m = 19.

Solution

Let n = depth of neutral axis.

$A_{st} = 4(\pi/4)(20)^2 = 1256.6$ mm^2

Taking moments of two area about NA, we get

$$bxnx\frac{n}{2} = mA_{st}(d - n)$$

Putting values, we get n = 213.5 mm

Lever arm a = d – (n/3) = 500 – (213.5/3) = 428.83 mm

Maximum bending moment

BM = $wl^2/8 = (12)(16)^2/8 = 54$ kNm= 54×10^6 N-mm

Let C be the compressive stress in concrete

$$M_r = \frac{1}{2}Cnb(a) = \frac{1}{2}C(300)(213.5)(428.8) = 13.732x10^6 C\ N-mm$$

M_r = External BM

∴ $13.732 \times 10^6 C = 54 \times 10^6$

∴ C = 3.93 N/mm^2

It "t" is corresponding stress in steel,

$$\frac{C}{n} = \frac{t/m}{d-n} = t = \frac{mC}{n}(d-n) = \frac{19x3.93}{213.5}(500-213.5) = 100.2\,N/mm^2$$

Example (SSC, Junior Engineer, 2010, Working stress method)

Determine the maximum superimposed distributed load which the beam section 220 mm x 440 mm (effective cover = 40 mm) reinforced with total area of tension steel 1256.64 mm^2, can carry, if the effective span is 5 m. Use M20 concrete and Fe 415 steel. Take m = 13.33.

Solution

Size of beam = 220 mm x 440 mm

D =440 mm d = 440 - 40 = 400 mm

A_{st} = 1256.64 mm^2

Effective depth, l = 5 m

Design constants

For M20 concrete σ_{cbc} = 7 N/mm^2

For Fe 415 steel, σ_{st} = 230 N/mm^2

m = 13.33

Critical depth of neutral axis (x_c)

$$x_c = \frac{m\sigma_{cbc}}{m\sigma_{cbc} + \sigma_{st}} xd = \frac{13.33x7}{13.33x7 + 230} x400 = 115.44\,mm$$

Actual depth of neutral axis (x_a)

$$\frac{bx_a^{\ 2}}{2} = mA_{st}(d - x_a)$$

$$\frac{220x_a^{\ 2}}{2} = 13.33x1256.64(400 - x_a) \Rightarrow x_a = 182.13\,mm > 115.44(x_c)$$

This section is over reinforced, concrete will reach upto maximum permissible stress prior to steel ($C_a = \sigma_{cbc}$).

Moment of resistance

$$MR = \frac{C_a}{2} bx_a \left(d - \frac{x_a}{3} \right) = \frac{7x220x182.13}{2} \left(400 - \frac{182.13}{3} \right) = 47.58\, kN - m$$

If uniformly distributed load of "w" is applied total load including self weight then

$$\frac{wl^2}{8} = 47.58 \Rightarrow w = 15.22\, kN / m$$

∴ UDL is 15.22 kN/m.

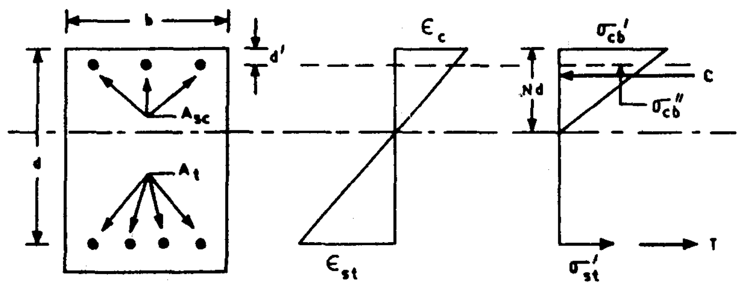
Problem (SSC, Junior Engineer, 2009, Working stress method)

A reinforced concrete beam 400 mm x 650 mm (effective) in section is reinforced with 3 bars of 28 mm ϕ. If the effective span is 5 m, find the concentrated load the beam can support at the centre. Assume M 20 concrete and Fe 250 steel (m = 13.33).

Answer: 110.276 kN

DOUBLY REINFORCED BEAM

See following diagram.



$$(bx^2/2) + (1.5m - 1)A_{sc}(x - d') - mA_t(d - x) = 0$$

Force of compression concrete

$$C_1 = bNd(\sigma_{cb}'/2)$$

Force of compression in steel

$$C_2 = (1.5m - 1)A_{sc}\sigma_{cb}''$$

$$\sigma_{cb}'' = \left(\frac{Nd - d'}{Nd} \right) \sigma_{cb}'$$

Force of tension in steel

$$T = A_t \sigma_{st}'$$

where A_{sc} is area of compression steel, d' is centre of gravity of compression steel from extreme fibre in compression, σ_{cb}" is compression stress in concrete at level of compression steel.

The force C_1 acts at a distance Nd/3 from top fibre. The force C_2 acts at a distance d' from top fibre. Their combined line of action acts at a distance "a" from the top fibre.

$$a = \frac{C_1 Nd/3 + C_2 d'}{C_1 + C_2}$$

Moment of resistance with respect to compression

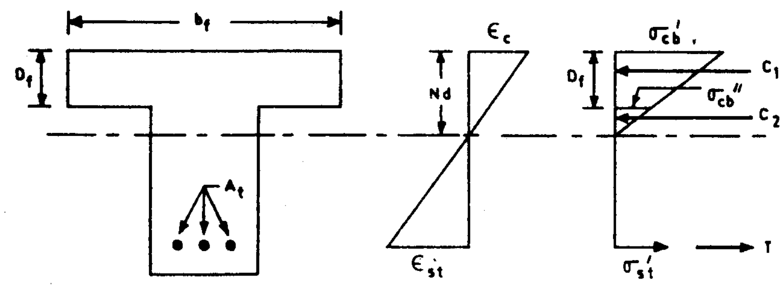
$$= C_1(d - Nd/3) + C_2(d - d')$$

Moment of resistance with respect to tension

$$= A_t \sigma_{st}'(d - a)$$

T-BEAM

See following diagram.



$$b_f D_f (x - 0.5D_f) + (b_w/2)(x - D_f)^2 - mA_t(d - x) = 0$$

Force of compression in flange

$$C_1 = b_f D_f [(\sigma_{cb}' + \sigma_{cb}'')/2]$$

Force of compression in web

$$C_2 = b_w(Nd - D_f)\sigma_{cb}''/2$$

Force in tension steel

$$T = A_t \sigma_{st}'$$

where b_f is effective width of flange, D_f is depth of flange, b_w is breadth of web, $x = Nd$ = depth of neutral axis which is assumed to lie in the web. σ_{cb}" is compressive stress at depth D_f from the top fibre.

The force C_1 acts at a distance a_1 from the top fibre

$$a_1 = \left[\frac{\sigma_{cb}' + 2\sigma_{cb}''}{\sigma_{cb}' + \sigma_{cb}''}\right]\left(\frac{D_f}{3}\right)$$

The force C_2 acts at a distance a_2 from the top fibre

$$a_2 = D_f + \left(\frac{Nd - D_f}{3}\right) = (2D_f + Nd)/3$$

Their resultant line of action lies at a distance "a" from the top fibre.

$$a = \frac{C_1 a_1 + C_2 a_2}{C_1 + C_2}$$

Lever arm will be (d - a).

Moment of resistance with respect to compression

$$= C_1(d - a_1) + C_2(d - a_2)$$

Moment of resistance with respect to tension

$$= T(d - a)$$

Example (SSC, Junior Engineer, 2013, Working Stress)

Design a cantilever beam which projects beyond the fixed end by 3 m. The superimposed load on it is 10 kN/m. Use M 20 grade (σ_{cbc} = 7 N/mm^2) of concrete and Fe 415 steel (a_{st} = 230 N/mm^2). Assume moderate exposure conditions.

Solution

Effective span (cantilever Beam) =3 m

Live load = 10 kN/m

Concrete grade = M 20

Steel grade = Fe 415

Assume, width, b = 300 mm

Overall depth, D = 600 mm

Self weight of the beam

= 0.6 x 0.3 x 1 x 25 = 4.5 kNm

Total load = 4.5 kN/m + 10 = 14.5 kN/m

Max bending moment = $wl_e^2/2$ = 14.5 x $3^2/2$ = 65.25 kNm

Design Constants

For M20 concrete and Fe 415 steel

m = 13; σ_{cbc} = 7 N/mm^2

$$k_e = \frac{m\sigma_{cbc}}{\sigma_{st} + m\sigma_{cbc}} = \frac{13x7}{230 + 13x7} = 0.283$$

$$J = 1 - \frac{k}{3} = 1 - \frac{0.283}{3} = 0.905$$

$$Q = \frac{1}{2}\sigma_{cbc} jK_e = \frac{1}{2}(7)(0.905)(0.283) = 0.896\, N/mm^2$$

Effective depth

$$d > \sqrt{\frac{MR}{Qb}} \Rightarrow d > \sqrt{\frac{65.25x10^6}{0.896x300}} \Rightarrow d > 492.6$$

Use clear cover for beam 25 mm, maximum diameter = 20 mm

Effective cover = 25 + (20/2) = 35 mm

D > 492.6 + 35 = 527.6 mm (required)

$D_{provided}$ = 600 mm > (D = 527.6 mm)

d = 600 – 35 = 565 mm

$$MR = \frac{1}{2}b(x)\sigma_{cbc}\left(d - \frac{x}{3}\right)$$

$$65.25x10^6 = \frac{1}{2}(300)(x)(7)\left(565 - \frac{x}{3}\right) \Rightarrow x = 118.23\,mm$$

$$MR = \sigma_{st}A_{st}\left(d - \frac{x}{3}\right)$$

$$65.25x10^6 = 230A_{st}\left(565 - \frac{118.23}{3}\right)$$

∴ A_{st} = 539.76 mm^2

Provide 5-12 mm diameter bars.

$A_{st(provided)}$ = 5 x (π/4)(12)^2 = 565.2 mm^2 > A_{st} (required)

Example (SSC, Junior Engineer, 2011)

Design a cantilever beam with a clear span of 3 in which carries a superimposed load of 15 kN/m. Its depth varies from 500 mm at the fixed end to 150 mm at the free end. Show reinforcement with a neat sketch.

Solution

Design constants and limiting depth

Assuming steel Fe 415 and concrete M20

f_y = 415 N/mm^2; f_{ck} = 20 N/mm^2; x_{ulim}/d = 0.48

$$Q_u = 0.36 f_{ck}.\frac{x_{u\lim}}{d}(1 - 0.42x_{u\lim}) = 0.36(20)(0.48)(1 - 0.42x0.48) = 2.759\,N/mm^2$$

Computation of design BM

Self weight = (1/2)(0.5 + 0.15) x 3 x 25000 x b

Assuming b = 250 mm leads to a self weight of 6093.75 N.

This will act at

$$\frac{0.5 + 2x0.15}{0.5 + 0.15}x\frac{3}{3} = 1.23\,m \text{ from fixed end.}$$

Superimposed load = 15000 N/m.

Moment at fixed end

$$M = 6093.75x1.23 + \frac{15000x3^2}{2} = 74995.31\,N - m$$

Design moment

$$M_u = 1.5x74995.31 = 112.492\,x10^6\ N - mm$$

Shear force "V" at the edge of support

$$V = 6093.75 + 15000 \times 3 = 51093.75 \text{ N}$$

Design shear force

$$V_u = 1.5V = 76640.62 \text{ N}$$

Computation of width

$$b = \frac{M_u}{Q_u d^2} = \frac{112.492x10^6}{2.759x500^2} = 163.09\,mm < 250\,mm$$

Take b = 200 mm

Steel reinforcement

$$A_{st} = \frac{0.5 f_{ck}}{f_y}\left[1 - \sqrt{\frac{4.6M_u}{f_{ck}bd^2}}\right]bd$$

$$= \frac{0.5x20}{415}\left[1 - \sqrt{\frac{4.6x112.492x10^6}{20x200x500^2}}\right]x200x500 = 735.78\,mm^2$$

Number of 10 mm diameter bars = 735.78/78.54 = 9.36 say 10 bars.

Actual $A_{st} = 10(\pi/4)10^2 = 785.4 \text{ mm}^2$

Curtailment of reinforcement

Since there are only two bars otherwise we could curtail one bar at x distance from free end because BM decrease towards free end upto zero.

Shear reinforcement

$$\tau_v = \frac{V_u - \frac{M_u}{d}\tan\beta}{bd} \quad where \tan\beta = (500 - 150)/1500 = 0.233$$

$$= \frac{\left(76640.62 - \frac{112.492x10^6}{500}x0.233\right)}{200x500} = 0.242 < \tau_{c,\min)}(0.28\,N/mm^2$$

Hence provide only nominal shear reinforcement

Spacing $S_v = \frac{2.175 A_{sv} f_y}{b}$

Using 8 mm ϕ 2 legged stirrups

$$A_{sv} = 2 \times 50.3 = 100.6 \text{ mm}^2$$

$\therefore$ $S_v = \frac{2.175x100.6x415}{200} = 453.70\,mm$

However maximum spacing = 0.75 d = 0.75 x 500 = 375 mm

S, provide spacing S_v = 300 mm

AXIALLY LOADED COLUMN

Short Column

The permissible axial load on a short column reinforced with longitudinal bars and lateral ties should not exceed that given by the following equation:

$$P = \sigma_c A_c + \sigma_{sc} A_{sc}$$

where P = permissible axial load on column, A_c = net area of concrete, σ_c = permissible stress in concrete in direct compression, σ_{sc} = permissible stress in steel in direct compression, A_{sc} = area of longitudinal steel

Slender Column

Strength of a slender column is equal to the strength of a short column multiplied by a reduction factor which is given as follows:

$$C_r = \left[1.25 - \frac{l}{48b}\right]$$

or

$$C_r = \left[1.25 - \frac{l}{160r}\right]$$

where l = effective length of column, b = least lateral dimension of column cross section, r = least radius of gyration of column

Online Support

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